Examining Teachers' Development in Representing and Conceptualizing Linear Relationships within Teaching Practice[1]

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Abstract. *This paper examines a range of data from four teachers who participated in eight, three-hour professional development sessions focusing on the teaching issues involved in conceptualizing and representing linear relationships. Drawing on data from pre-post measures, interviews, and classroom observations, we will examine the teachers' development of content and pedagogical content knowledge in relationship to changes in their practice – both in how they think and talk about their practice as well as what they actually choose to do. This preliminary survey study is an attempt to relate various assessments and to gather initial evidence as to what teachers apply in practice in order to begin to identify and define some of the issues to consider in framing future efforts aimed at assessing impact on practice.*

Acknowledgements. The authors wish to acknowledge the contributions of several individuals in helping to shape the work of this project. Dr. Nicholas Branca, a project PI, prepared the mathematical commentaries and helped define the mathematical goals. Throughout the VCMPD project, the mathematical learning goals were influenced by our work with Deborah Ball, Hyman Bass, Mark Hoover and the other members of the Mathematics Teaching and Learning to Teach group at the University of Michigan. Drs. James Stigler, Alfred Manaster, and Ramesh Gangoli each contributed significantly to these efforts. The authors would like to express deep appreciation to the VCMPD videotaped teachers (Debbie Alteneau, Cindy DeClerq, Gisele Nassif, Kirk Walters and Maryann Wickett) for contributing their teaching practice to the profession in an effort to improve mathematics teaching and learning in this country.

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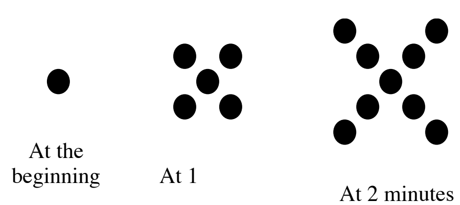
Introduction

The need for professional development for teachers of mathematics has never been greater. Simply providing more professional development, however, is not the point. Professional development is of no value if it does not result in improved practice. Strikingly absent have been wide-spread efforts to assess learning from professional development. While some have attempted to measure teacher learning from professional education experiences (Stein, et. al, 2003; Smith, et. al, 2003; Barnett and Tyson, 1993; Franke, Carpenter, et. al, 2000; Fenemma, 1992; Shifter, 1998), most evaluation efforts to date have focused primarily on attempts to assess teacher knowledge and beliefs. There have been some efforts to also gather self-report data on what teachers plan to use in practice, but self-report data is often suspect. The relationship between the characteristics of the professional development materials, how they are used, and what teachers learn requires careful research before one can make claims about their effects (Wilson and Berne, 1999). This paper attempts to examine a range of data from four teachers to better understand what teachers might learn and use in their practice. This is only a preliminary survey study to begin to define what data might be useful in a more rigorous effort to address these questions. This small-scale effort was an attempt to relate various assessments and to gather initial evidence as to what teachers apply in practice in order to begin to identify and define some of the issues to consider in framing future efforts aimed at assessing impact on practice.

In Fall 2002 a group of eleven middle school mathematics teachers participated in a course consisting of eight 3-hour after-school sessions of professional development aimed at exploring the teaching issues involved with conceptualizing and representing linear relationships.

The first and last sessions of this professional development module involves teachers in analysis of the following five-minute “Growing Dots” video clip from a 9th grade mathematics classroom (see figure 1 below).

Kirk, a 9th grade Algebra teacher, poses the following task to his students with the goal of helping them visualize and conceptualize slope and y-intercept:



Study the sequence of dots. Describe the pattern you see. Assuming the sequence continues in the same way, how many dots are there at 100 minutes? Find an equation for the number of dots at *t* minutes.

After working on the problem for a few minutes, Kirk brings the class together to discuss their solution methods. Danielle shares her answer of x • 4 + 1 *and Kirk asks her to illustrate how she connected her expression to the dots at the board. When Kirk asks for a different method, James shares his method*

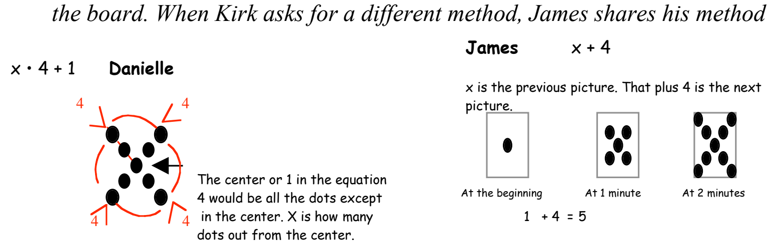


Figure 1

This video offers the opportunity for teachers to consider the multilayered complexities of mathematics teaching. Peeling back the layers of what Kirk (the teacher) was faced with in the above five-minute instance of practice reveals some of the intellectual demands he encountered in his work. Among other things, he was faced with:

- *Deciding whom to call on and in which order*
- *Dealing with how to get Danielle and James to communicate and represent their methods clearly to the whole class*
- *Evaluating the mathematical logic of Danielle and James' thinking on the spot*
- *Recognizing and reconciling the explicit and recursive methods presented*
- *Figuring out what to do with these two methods in relationship to his goal of enabling students to conceptualize slope and y-intercept*
- *Simultaneously assessing where the whole class was in relationship to his goals and to these methods and representations*

Magdalene Lampert (2001) has characterized these as "the problems in teaching." The challenge for teacher educators is to help teachers effectively address these "problems in teaching" in ways that create powerful opportunities for all students to learn worthwhile mathematics.

As teachers attempt to unravel and make sense of this portrayal of the demanding work of teaching what do they learn and what do they apply in their practice? Do they deepen their understanding of mathematics—the mathematics useful and usable for teaching? Are they developing increased capacity to enact lessons? Do they apply their knowledge to improve their teaching?

This paper traces four teachers' growth in understanding of conceptions and representations of linear relationships as a result of their engagement in the eight VCMPD sessions. Drawing on data from the embedded and external pre-post measures and pre-post classroom observations, this paper examines the teachers' development of content and pedagogical content knowledge in relationship to changes in their practice – both in how they think and talk about their practice as well as what they actually choose to do.

The VCMPD Materials

The Videocases for Mathematics Professional Development (VCMPD)[2] Project engaged in the research and development of videocase curriculum as a tool for mathematics professional development. The goal of the VCMPD Project was to design and develop professional development materials using real video images of mathematics classroom to help teachers: (1) develop a more robust understanding of student conceptions of linear relationships; and (2) improve their capacity to prepare and enact tasks that enable students to develop conceptual understanding and representation of linear relationships. Through strategic design and scaffolding, the videos and the accompanying materials are intended to help teachers, grades 5 – 10, deepen their understanding of both mathematics content and classroom pedagogy.

The first module, *Conceptualizing and Representing Linear Relationships,* is a sequential series of eight 3-hour sessions that are designed to enrich teachers' ability to teach about linear relationships and deepen their own detailed knowledge of the distinctions and linkages among the various representations. Each session has at its core one or two digital video clips of a mathematics classroom. These clips are unedited segments selected from real classroom footage of un-staged mathematics lessons, representing a range of grade levels, geographic locations and student populations.

A central element of each session is an opportunity for participants to collectively explore the mathematics, student thinking, and teacher decisions encountered in the video. Other elements of

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sessions include related readings or research as well as a linking to practice component that is designed for teachers to connect and integrate their professional development experience into their own teaching practice. Over the course of the sessions some classrooms and problems are revisited, maintaining the theme of earlier sessions while considering variations. Lesson transcripts, detailed lesson graphs[3], commentaries, and categories of solution methods/representations are also provided. Software developed by LessonLab[4] makes it possible to readily access any moment in the video and to display subtitles while viewing.

An underlying assumption of the design of these materials is that facilitating groups of teachers is difficult and complex work. These materials are predicated on the belief that teachers need opportunities to work together in settings to socially construct and reconstruct their understandings of teaching and learning of mathematics. Just as the teacher in the classroom pays attention to students' learning, these materials require a facilitator paying attention to teachers' learning- they must know the materials, their intent and design, and take charge of structuring the learning experience for others. Establishing a community of learners in this process, where inquiry is valued, presents challenges. The facilitator must learn about her participants and honor what they bring to each experience. Materials have been developed to support the facilitator in this demanding work. The facilitation guide contains such information as: a complete overview of the materials, explanations and rationale of the underlying principles and specific goals, sample agendas and guidelines for sessions, lists of references and useful resources, tips for facilitation, mathematics commentaries and excerpts from a composite facilitator's journal chronicling the experiences of others having used these materials.

Figure 2 represents the sequence of experiences in the module.[5] The columns represent the eight - sessions in the module and the suggested sequence of activities which varies from session to session.

Each shape represents a particular kind of activity described below:

Situating the work (circles). This provides opportunities to orient and situate the activities and learnings in the context of mathematics education. The purpose is to help participants make explicit their own theories on mathematics teaching and re-examine and refine those theories.

Mathematics (octagons). Participants engage in the exploration of the same mathematical tasks as those portrayed in the videos. The purpose is to develop an understanding of the mathematical demands of the task in order to be able to interpret students' thinking and teacher decisions. In doing so, participants also deepen their own mathematical understandings.

Video and Discussion (rectangles). This is at the heart of the session, but it does not stand alone. Here the purpose is to use a concrete, authentic piece of teaching practice to examine the teaching and learning of mathematics. The video experience is intended to engage teachers in thinking and reasoning about mathematics teaching. Participants are asked to examine the video from an analytic frame—to "get into" the possible mathematical thinking of the teacher and the students and to investigate these and the mathematics from several different perspectives.

Compare and Contrast (triangles). This provides focused opportunity to compare and contrast the mathematical ideas, the mathematical tasks, the solution methods and the teaching practices

[3] The "lesson graph" was originally developed by Nanette Seago to show graphical representations of the various "cultural scripts" that Stigler and Heibert (The Teaching Gap) found in the TIMSS Videotape Study of US, German, and Japanese Teaching. She has further developed it in a more detailed version for use with videocases.
[4] For more information about the software used, contact Lesson Lab, Inc. at www.lessonlab.com or 3300 Ocean Park Blvd., Santa Monica, CA 90405, 310.664.2300
[5] This representation was inspired by Margaret Smith and her work in designing pre-service courses.

across sessions. The purpose is to develop a deeper understanding of the core module goals by comparing and contrasting the same set of ideas across multiple and varied contexts.

Linking to Practice (pentagons). These activities attempt to bridge between the session goals and teachers' own practice. These "linking to practice" activities are attempts to provide opportunities to apply their new ideas to their own teaching situation and reflect on its meaning.

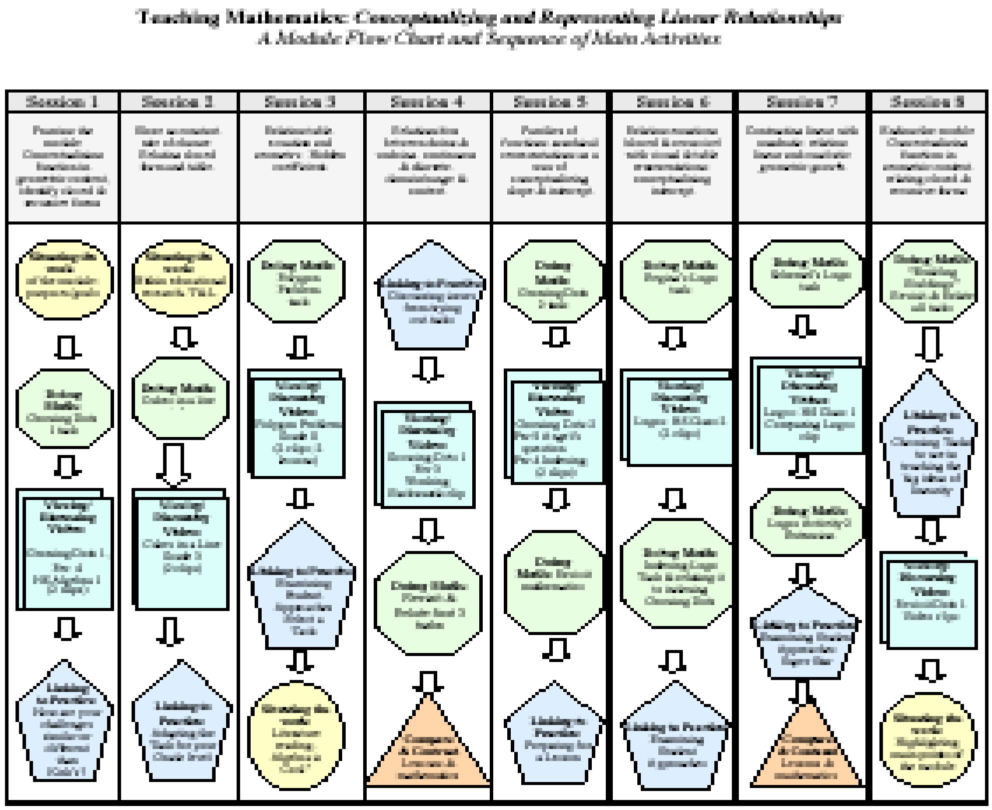

Figure 2

Goals for teacher learning

The VCMPD materials have the goal of helping teachers develop the knowledge, skills and sensibilities to reason and make informed decisions about their own teaching of mathematics—specifically linear relationships. This means that teachers deepen their understanding of the mathematics that is useful and usable in teaching the mathematics of linearity. Because mathematics teaching demands a different kind of knowledge—differing from what is typically experienced in university mathematics courses—these materials attempt to develop mathematical knowledge that teachers will find useful and usable in teaching (Ball (1991); Ma 1999; Ball & Bass 2000). We know that teachers participating in this curriculum will bring varied and important experiences to bear on their learning. Given these differences, the exact nature and depth of what they might learn will vary. We do however, define a core set of learning goals for all teachers and also indicate additional (supplementary) learning opportunities these materials might afford. Three categories of mathematical knowledge constitute the overlapping learning domains within which these goals reside. The core (C) and supplementary (S) mathematical learning opportunities are described for each.

1. Disciplinary knowledge

- Robust and flexible understanding of linear relationships which involves:
  - Linear functions -starting point (and the effects of shifting it), constant rate of change, contrast with non-linear (C)
  - Fluency in relating various representations - tabular, symbolic, graphical, pictorial, recursive and closed (C)
  - Continuous versus discrete relationships (S)
  - Relationships to proportional reasoning (S)

2. Students' mathematical knowledge

- Understanding what students need to know about linear relationships which includes:
  - The relationship between rate of change and slope (C)
  - Relationship between recursive and explicit approaches (C)
  - Relationship among starting point, constant term, y-intercept and its representation (C)
- Understanding how students develop conceptual understanding of linear relationships which involves:
  - Identifying common student conceptions and misconceptions (C)
  - Students' learning trajectory over time (S)

3. Curricular (Instructional) knowledge

- Ability to choose, prepare and enact tasks to enable students to develop an understanding of linearity, which involves:
  - Predicting student methods and reasoning (C)
  - Interpreting, distinguishing, categorizing and relating student methods for expressing/representing/solving linear functions in geometric contexts (C)
  - Choosing, using and relating various mathematical representations - table, graph, symbolic (C)
  - Recognizing how the context helps/hinders the development of conceptual understanding of linearity (C)
- Understanding the role of linearity in the curriculum which involves:
  - Knowing where linearity resides in the curriculum (S)
  - Knowing how linearity is connected to other areas of mathematics (S)

In addition to learning the mathematics for teaching linearity, the VCMPD materials have goals related to teachers' professional practice. They aim to inculcate a set of norms for learning about teaching. These norms are aimed at developing teacher generativity[6]—recognition that they are professionals and can continue to learn from and through their own practice. These goals include:

- Building a language of practice to communicate, reason and talk with precision about teaching;
- Developing a questioning attitude within and outside their classroom;
- Developing the habits of inquiring into practice, envisioning alternatives and extracting from complexity.

[6] Generativity is defined by Franke and Carpenter (2001) as individuals' abilities to continue to add to their understanding; apply their knowledge to learn new topics and solve new and unfamiliar problems.

The goals for these materials are ambitious. To what degree teachers achieve these goals, and what constitutes evidence of movement toward them, will be considered in the following sections of this paper.

Project Evaluation Efforts

Researching what teachers were learning from their experiences with the VCMPD video-based curriculum while simultaneously developing materials presented challenges. As Wilson and Berne (1999) note “studying the phenomenon while one creates it always presents particular problems because one’s attention must be bifocal: creating meaningful professional development and doing rigorous research”. How do you study what teachers learn and use when you are continually revising and refining the materials? Yet it is imperative that the course materials contribute to teacher growth, since improving teaching was the point in developing these materials. We needed to determine whether these materials, when used as designed, result in teacher learning. The choice was made to study the VCMPD staff pilots of the developers and five highly supported pilot sites rather than a larger set of distant field test sites that had little interaction with the developers in order to assess this impact.

The first module, *Teaching Mathematics: Conceptualizing and Representing Linear Relationships*, was piloted in five different sites in the country. Three different outside evaluation efforts assessed various aspects of this work. Two independent efforts were aimed at assessing teacher learning from these materials and one examined facilitation. Heather Hill and Rachel Collopy examined what teachers learned from these materials. They developed an “external” assessment that attempted to measure growth in teachers’ content knowledge and pedagogical content knowledge using pre and post measures (Hill & Collopy, 2003). We use the term “external” because it is not an assessment that is not part of the designed work within the module—it is separate from the professional development experience. The instrument Hill and Collopy developed was based on their work on the “Study of Instructional Improvement”[7] and “Developing Knowledge for Teaching Mathematics, ” studying the professional development institutes supported by the University of California, Office of the President (Hill & Collopy, 2002). The assessment consisted of two parts and required teachers approximately one hour to complete. Part I consisted series of questions asked teachers about their how they plan for, enact and reflect on their teaching. Part II, the more time-intensive portion, consists of nine mathematics-demanding tasks embedded in a context of teaching. Within these 9 tasks were 14 multiple-choice possibilities and 7 open-ended items; of these 7 open-ended items, four could be scored correct/incorrect. Ten of the multiple-choice items were drawn from an item pool developed by the Study of Instructional Improvement/Learning Mathematics for Teaching (SII/LMT) projects (Ball & Hill, in preparation; Hill, Schilling & Ball, 2002). These were piloted with a large sample of teachers participating in California’s mathematics Professional Development Institutes (Hill & Collopy, 2003). An example of one of the open-ended tasks is illustrated in Figure 3.

7. Ms. Hernandez's class was looking at the following table one day. She asked her students to think of a way that would help them find *x* for any given *n*, without having to continue the table.

n	*x*
1	1
2	3
3	5
4	7

One student wrote "The answer is x + 2." Mrs. Hernandez was about to mark this student's work as wrong when Mrs. Johnson, her student teacher, said she had a guess about what the student might have been thinking. In <u>your</u> opinion, what might this student have been thinking?

Figure 3

Horizon Research Inc., under the guidance of Iris Weiss and Dan Heck, has developed an "embedded" assessment to measure the impact of the program on teachers' content knowledge and pedagogical content knowledge. Embedded means that the instruments are part of the actual work teachers do within sessions—it is *embedded* in the professional development experience. One of the embedded assessment instruments asked teachers to solve the mathematics task, reflect on their own approaches to solving the task, and predict approaches students might use to solve it. Near the end of the module they were asked to do a similar process with another mathematics task. There was also a pre and post video analysis task. Individually participants watched a video clip and were asked to select a moment or interchange that was important or interesting, then write an explanation as to why. One embedded task asked participants to analyze student work samples and indicate what they might do next in this class if these papers were representative of the class as a whole. The embedded assessment permitted pre-post comparisons for analysis of impact on teacher learning of content knowledge and pedagogical content knowledge in relationship to the goals of the module. While these measures were less intrusive to the professional development experience, there were many challenges in using these types of open instruments to make reliable and valid claims about teacher learning. In addition, a comparison group of similar teachers responded to both the embedded and external pre-post tasks, which allowed treatment group-control group comparisons.

Inverness Research Associates conducted a study of the use of the VCMPD materials in the five pilot settings located across the country—how did facilitators use these materials, what supports did they need and what did teachers seem to be making of their experiences with them? They identified the benefits and challenges of using the materials and the supports required by facilitators to use these materials effectively. To gather data they used multiple interviews with facilitators, interviews with session participants, and observation of different sessions at each of the pilot sites. Inverness Research found that overall the materials were feasible for use by local facilitators in real settings and that every facilitator judged the materials to be of high quality, thoughtfully designed and coherent. Facilitator training was found to be critical to the level of success of the professional development in these pilot settings, and yet the importance and difficulty of "mining the video" surfaced repeatedly (Tambe, St. John, et al, 2002). Data from this evaluation were not collected as part of this study.

With regard to the external and embedded assessments, early analysis from pilot efforts suggested that teachers can gain in the ability to "see" and "hear" alternative approaches to solving problems involving linearity and to link various representations of linear relationships. In addition, teachers appeared to deepen their knowledge of concepts such as slope and y-intercept.

The Study

During the Fall of 2002 Nanette Seago, Project Co-PI, conducted a pilot of the primary module (*Teaching Mathematics: Representing and Conceptualizing Linear Relationships*) with eleven teachers (9 women, 2 men from grades 6-8) from three middle schools in a large urban district. Teachers volunteered to attend eight three-hour sessions where they participated in project seminars. Sessions were conducted weekly at a district center after school over a three-month period, and each session was videotaped.

The eleven teachers also attended a two-hour pre and post session for data gathering purposes. Each teacher completed both the external and embedded assessments. A control group of 7 teachers (volunteers for participation in future sessions) were administered both pre and post assessments. Teachers in both the pilot and control groups were paid a small stipend for their participation in data gathering. An additional open-response evaluation was administered during the last session asking teachers what they feel they gained from the sessions and how they planned to use what they learned.

Four of the eleven teachers from this pilot group constitute the subjects of the study reported in this paper. Data from their pre and post external and embedded assessments were analyzed as part of this study along with additional data gathering efforts described below.

Observations. The four study teachers volunteered to participate in observations and interviews. No attempts were made to randomize the volunteers given the small group size and selection was based on who was available. As it was, two teachers from two of the participating middle schools volunteered. Two were teaching grade 6 (Barbara and Debbie[8]) and two grade 7 (Arlene and Charlene). Each of the four teachers was observed for one class period prior to the second videocase session and then again a few weeks after the last session. Each observation included a pre-lesson interview conducted by telephone, typically the day prior to the observed lesson. The pre-interview included a set of questions to determine background information on the class and the intended purpose of the lesson. Following the lesson the teacher was again interviewed using a set of questions to elicit teacher reflection on her lesson and next steps for the class. The same set of interview questions were employed for both pr- and post-observations.

The observations were conducted using an instrument modified from the classroom observation protocols developed by Horizon Research, Inc. for the Local Systemic Change Initiative. Project co-PIs modified protocols to attempt to reflect the mathematical focus and goals of the module materials. Lessons were evaluated across four sets of lesson criteria: design, implementation, mathematical content, and classroom culture/mathematical norms. A 5-point likert scale (from "not at all" to "to a great extent") was employed with "don't know and not applicable as alternate choices. A five-point synthesis rating was given for each. The scale ranged from not at all reflective to extremely reflective of the goals of VCMPD or worthwhile mathematics. In addition, a 5-point likert scale (from negative effect to mixed or neutral effect to positive effect) was used on ten criteria reflective of the lesson's likely impact on students' understanding of

[8] Pseudonyms are used for each teacher

mathematics. A final capsule description depicted the overall quality of the lesson. Following the Horizon LSC scheme, five levels from ineffective to exemplary were identified.

An independent observer conducted each observation. The non-teaching project co-PI accompanied the observer on the first two observations to calibrate the scoring. The observer was deemed a highly capable and reliable observer—an experienced mathematics educator, familiar with the project goals and former associate of project PIs. She independently conducted the interviews and observations using the prescribed protocols.

Video Analysis. The videotapes from the first and last session were examined for any changes in how the 4 teachers engaged in discussion.

Post 3-month Interview. Three months after the last session the study teachers were sent copies of their own pre and post embedded and external assessments and asked to consider whether they felt these assessments captured what they learned. Specifically, they were asked to consider: What do you think these say about what you took from these sessions? What don't they say? What else should we have asked that might better reflect what you gained or didn't gain from the experience? An informal telephone interview was conducted by the non-teaching co-PI seeking their response to these questions. In addition, they were asked to reflect on their experience with the module—how it has impacted (or is impacting) their teaching and what they think other teachers might gain from participation in a similar series.

The following data was assembled and analyzed for each of the four teachers:

- Pre and post external assessment data
- Pre post embedded assessment data
- Open response survey at end of last session
- Pre and post observation interview notes from beginning and post-session observation
- Beginning and post-session observation protocol data
- Notes from post 3-month interview

Findings

Each of the four teachers participated in all of the VCMPD sessions and in all data collection efforts, with the exception that two of them (Arlene and Debbie) were unable to participate in the post 3-month interview. A summary of the various data gathering efforts is below.

First, teachers were asked as part of the pre survey, "Can you tell us why you're here?" The four teachers gave slightly different reasons for participating in the videocases

Arlene: *It was recommended by Mary, I had no idea what it really was.*

Debbie: *To grow professional and learn more strategies for the classroom.*

Barbara: *To assist new teacher on campus – also I love math and I am excited about any and all information to expand my experience.*

Charlene: *Never heard of anything like it before and thought it would be interesting if I could extend my professional growth in math.*

They were also asked about their teaching history. Their responses are in Table 1 below.

Teacher	Grade(s) they currently teach	Grade levels taught in the past	Number of years taught
Arlene	7	5 and under	4
Debbie	6	5 and under	3
Barbara	6	7, 8	6
Charlene	7	7, 8	12

Table 1

What did they say they learned?

At the end of session 8 teachers were asked to write a response to, "What did you learn from your experience about mathematics? About teaching mathematics?" This is reported to provide a comparison with the external and embedded assessments. Additional self-report data are found later in this section.

Arlene: *There are so many approaches – seemingly simplistic that lead to very sophisticated conclusions. Most of the approaches simplified to the same equation. So many students stop, if they believe their equation is incorrect. How many would find their answers were correct, but approached differently if allowed fruition?*

Debbie: *Mathematics is very exciting and FUN! Most important I learned that in teaching mathematics it is important to allow student to discover solutions, give them time to find the solutions. Also, pay close attention to how students approach their solutions – there are usually many methods (ways) in which to solve a problem, whether it be closed, explicit, recursive.*

Barbara: *What learned is collaboration is the best way to learn. Usual representation also important, class discussion and time to absorb contribute to concrete understanding.*

Charlene: *From this experience I have learned that mathematics is represented and conceptually processed by individuals in so many ways. As a teacher I'm already aware of the different learning styles of my students, however, through this experience I am more aware of the differentiation in conceptualness. From this experience I have learned to incorporate more student led explanations ("teaching") while teaching to see where the gaps are in their learning of a particular skill.*

External Assessment

This assessment involved 9 tasks, with 14 possible correct answers in the multiple choice portion and 4 possible correct answers in the open-ended portions. The multiple choice and two of the open-ended required teachers to provide a solution to a linear functions task. Several of the tasks also asked teachers to provide interpretations relating to instruction. The results from the four study teachers are in table 2 below along with those from the entire 11 pilot teachers and the comparison group.[9]

[9] data on entire pilot group from Hill and Collopy, 2003

	Arlene	Debbie	Barbara	Charlene	Avg.	Pilot	Comparison
Pre multiple choice	57% (8)	86% (12)	50% (7)	64% (9)	75% (9)	79% (11.1)	79% (11)
Post multiple choice	100% (14)	86% (12)	86% (12)	71% (10)	86% (12)	83% (11.6)	77% (10.8)
Pre open-ended	3	3	0	3	2.25	1.9	2.5
Post open-ended	4	4	3	3	3.50	3.4	2.8

Table 2

The four study teachers appear to be representative of the other pilot group members on the pre/post measures with the pre open-ended being slightly higher. Their results on the pre are close to those of the comparison group on the pre and exceed those of the comparison on all of the post measures. The sample size is too small to make any conclusions that would hold up under statistical scrutiny. For each of the study teachers we also examined their responses that implied instructional implications.

<u>Arlene</u> improved considerably on her pre/post assessment. She demonstrated that there was limited change, however, with regard to instructional implications. For example, on task #7 teachers were asked what a student might have been thinking when he answered x + 2. On the pre she said, "they were looking at the table vertically, instead of horizontally." On the post she indicated, "the student was looking at the difference in the x values – not the relationship between x and n. It works for the following x."

<u>Debbie</u> demonstrated limited improvement pre/post. She even answered one incorrectly that was correct on the pre. On the pre she had little to say on the open-ended items related to instruction, whereas on the post she was able to provide more detail related to instructional implications. On #7 on the pre she indicated, "The student was finding the pattern for x (reversed the variable)." On the post she said, "The student noticed that in the x column the number increased by 2. Had the student substituted to check their equation they would have noted their equation did not work. The equation should be 2x-1."

<u>Barbara</u> improved her pre/post multiple-choice and open-ended items. On the pre she provided no or incorrect explanations or examples related to instruction nor was little provided on the post. On question #7 on the pre survey she stated, "Instead recognizing the pattern growing by one, the student looked at the #4 = n & 7 = x as the difference being 2 as the difference being 2 as the function for the pattern." On the post she stated, "that the difference between the # is 2 the student should looked at the relationship between n and x. What happens to n to give me x?'"

<u>Charlene</u> showed only a small improvement on the multiple-choice and no change on the open-ended items. Her responses related to instruction were very similar pre and post. She did provide examples on both pre and post. On question #7 on the pre she stated, "the student may have been thinking that the pattern is increasing by 2 in the x column." On the post she stated, "in my opinion, the student saw in the table that the x values are each time being increased by 2."

Embedded Assessment

Mathematics tasks. Three tasks were part of the embedded assessment. The Growing Dots 1 task was considered the pre assessment, done at the beginning of session 1. The Logos task was part of session 6 and Buildings Task in session 8. All involved generalizing a linear relationship from a geometric context. The VCMPD materials placed emphasis on understanding a variety of representations (e.g. tables, visual[10], graphs, symbolic) and relationships among them. The teachers were asked to solve it in their preferred way, to predict how students might approach it that would likely lead to a correct solution, how they might approach it that would likely lead to an incorrect solution and what mathematical ideas the task offers the opportunity to teach students.

Arlene utilized a table as her preferred approach on each of the tasks. She did not provide a generalization for session 1 task, but did provide correct generalizations for sessions 6 and 8. She provided no evidence of a visual approach on any of the tasks either for her preferred method or predicting how students might approach the task. In predicting what students might do that would lead to a correct solution she actually was less detailed over time. In session 1 she wrote, "yes, although time consuming, the solution could be graphed. The table could be completed to 100. The student could also draw the dots adding 4 each time." In session 6 she wrote, "My students would find this very difficult." And in session 8, "T chart – I am honestly not sure. I think they would work back from building 3."

Debbie used a table to find a correct generalization on session 1 and 6, and used a visual method in session 8. In predicting what students might do, she too lacked much detail. In session 1 she wrote, "Yes, guess and check; draw more pictures." In session 6 she wrote, "Yes, pictures, trial and error" and "Yes, students might use a table to solve this problem or perhaps use trial and error (guess and check)" for session 8.

Barbara appeared to use a table to produce correct generalizations in all 3 sessions. In predicting what students might do, she exhibited some growth in rationale over time for what students might do. In session 1 she wrote," Possible solution to the problem – pictures, acting out." In session 6 she wrote, "Many ways: 1) continue drawing tiles, 2) generalize formula, 3) graphing slope" and in session 8 she indicated that they might produce a correct solution, "if they used a function table looking at the relationship between the # of buildings and the squares."

Charlene used a table to produce correct generalization for each. In predicting what students might do, she exhibited some growth in rationale over time for what students might do. In session 1 she said, "Yes, students may try and do a table calculating each min. until they get to a hundred." For session 6 she drew a table identical to her own approach and for session 8 she failed to provide an answer.

Interpreting student work. This task involved asking teachers to look at 4 student papers on a task similar to those in the videos. They were told to assume that these papers were reflective of the work of students in the class and based on that asked to give an assessment of the class's current level of accomplishment. They were also asked to "describe what you would do, what you would have the class do and what questions you would ask in the next lesson."

Arlene appeared to have a number of ideas with regard to what she would do next, "I would go back to the dot problem. I would color code the constant. Hopefully, this would allow them to

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[10] visual ways of seeing and representing functions was foreign to most secondary teachers

see that this is added to the function and why. The fact that the dots are distributed equally each minute would allow them to see the change is coefficient +4 each variable minute. This (I think is more usually - the dots – concrete.)"

Debbie stated what she would do next is, "I would use a different model for the students to determine a general equation – using blocks (manipulative). I would go over using a T chart and graphing the equation to see the linear pattern. I would emphasize what is the role of change and the constant. I would have students put numbers in the variables (substitute) to check the formula for accuracy."

Barbara indicated that, "After students have complete the project I would have students continue with more in depth conversation comparing the difference between each work sample. Next lesson model different approaches incorporating graphing. Emphasizing slope. Something in that direction. I believe that extending two function showing the change can be better explained through a graph."

Charlene wrote, "I would have my students graph their answers to see if their answers resulted in a linear pattern. For the next lesson I would give them another pattern (logos) and have them focus on what parts of the equation represent in relation to the logo. Perhaps, give them manipulatives to work with as well."

Analyzing Video. After watching the first video clip in session 1, participants were asked to individually write their reflections. The question asked them to "Select **one** moment or interchange within the following time segments that you find mathematically important/interesting and that demonstrates the teacher and/or students thinking about the mathematics of this problem." Four choices were provided, each with potentially important or interesting mathematics. They were then asked to write to 2 questions: What about this moment/interchange makes it mathematically important/interesting? Describe what the teacher and/or students might be thinking about the mathematics of this problem at this moment or during this interchange. The same video clip is shown in session 8 with teachers asked to respond to the same prompts. Session 1 and 8 are presented in table format for each teacher below.

Arlene.

Session 1	Session 8
What about this makes it mathematically important? Brandie said the 1 in the middle never changes. "Great time for constant, coefficient, and variable discussion. What do each of these terms mean in this equation?"	**What about this makes it mathematically important?** This was the perfect moment to clarify the differences and the roles of a variable, constant and coefficient. James used the variable to represent the first picture. A variable can have one or more numerical representation. Take James back to the 1st picture and ask him if his "x" is representing an unknown or a substitution. If "x" represents a number, looking at the 1st picture, what does it represent? Could the same number (1) or dot be seen in every picture? Does that mean it is a variable or a constant?
Describe what the teacher and/or students might be thinking. Teacher is trying to catch up with the amount of	**Describe what the teacher and/or students might be thinking.** What is the change from picture to picture? Does

different perspectives and questions and address each. Some of the students were frustrated because the perspectives have not come together for a coherent solution.	the given equation fit James' perspective? Is it the same as Danielle's or different?

Debbie.

Session 1	Session 8
What about this makes it mathematically important?	**What about this makes it mathematically important?**
I found it interesting that James did not want t include the center dot because it was not changing. It is important because it is the constant. Side bar: James did not want to go to the board because he feared failure. Important to build a secure/safe-learning environment.	The interaction between James & Kirk when Kirk ask why he didn't count the center dot was important/interesting. When ask why? Brandie (18:09) said "the center is not growing". It was unclear to her that the original question was to write an equation for any situation (to include all dots). Secondly, James was unclear about equations because he came up with add 4 to the previous one.
Describe what the teacher and/or students might be thinking.	**Describe what the teacher and/or students might be thinking.**
The teacher needs to get across that you need to include the 1 in the center.	Brandie interpreted the problem as what is the equation for the growing part of the dots, therefore, not adding in the 1. James not clear about variable or constant. He never came up with 4x, he just kept adding 4.

Barbara.

Session 1	Session 8
What about this makes it mathematically important?	**What about this makes it mathematically important?**
Danielle ability to see the constant being the center and the outside growing by 4. Recognizing that there are 2 things the center being (1) and growth by (4).	"Cause the center is not growing, it's just what's growing around it." This statement proves the center stays the same and is the constant. Although they do not count the center, I think that they are building on the concept. It becomes a teachable moment. Both James and Brandy recognize the growth pattern but don't know how to connect the center.
Describe what the teacher and/or students might be thinking.	**Describe what the teacher and/or students might be thinking.**
The student presented the information clearly and to the point. Danielle understood function and how it effected growth of shape.	Teacher seems to be clarifying about the growth pattern and wonders why and what the answer is and why they are not including the constant. James and Brandy know the center stays the same and that Danielle did include in her response, but didn't include or change her answer.

Charlene.

Session 1	Session 8
What about this makes it mathematically important? The point in which James explains that he didn't have a representation for the center dot or that, in other words, he really didn't think of the center dot as one.	**What about this makes it mathematically important?** James not realizing or wanting to count the middle circle is mathematically important. However, he does realize that the center is not growing – it's a constant.
Describe what the teacher and/or students might be thinking. I think James is thinking that the middle dot never changes it isn't expanding so he doesn't need to count it.	**Describe what the teacher and/or students might be thinking.** I think that James is thinking that because the center has already been counted initially from the beginning point to the 1 min., mark, whereas you get 1 + 4 = 5 at 1 min, then 4 dots are being added to that which gives you 9, he doesn't have to count or consider the one. He just wants to use repeated addition of 4. This is why he came up with multiplying 100 x 4, getting 400 instead of 401.

In addition to their written responses to the embedded prompts, the video discussions from sessions 1 and 8 were examined. There was somewhat of a shift noted in the degree of definitive versus tentative talk. For example, in session 1 Arlene stated, "I didn't like the first one [explanation] because I could tell she could think at a fairly abstract level and she is not able to articulate it just yet and when I have a student that does that it frustrates the dickens out of the rest of the class. I can understand what she's thinking but she's not able to articulate it. Kids aren't getting it and all it does is throw them into further confusion." I session 8 her comments reflect a somewhat more questioning tone, "When James labeled the first dot 'x' I really wish we could have asked James why did you label the first dot 'x'? What is another way looking at it you could have named it?" Following this, however, she goes on to say how she would lead him through a series of questions to "see" the right answer.

Debbie and Barbara were for the most part silent in session 1, offering no questions or comments of their own. In session 8 they both engaged in the discussion, primarily to raise questions such as, "Are you saying amount of growing dots or amount of total dots?" and "Doesn't [sic] Brandie and James understand to some degree what Danielle did? They repeated it. They repeated but still didn't get it?"

Other differences were noted in the way the teachers engaged across the sessions. In session 1 many people talked over one another—it appeared that they were trying to make their point and not listening to what other were saying. By the last session most of the teachers were listening intensively to each other, often building their comments and ideas from others. They referenced each other's ideas in their comments frequently. In addition, they were backing up many of their claims with evidence and reasoning.

Applying to practice

At the end of session 8 teachers were asked to write their responses to questions related to what they were using or intended to use in their practice.

Arlene reported, “I will spend more time investigating and encouraging my students to develop their approach. More often than not, they are correct and need only a nudge. The videos showed me that we don’t relate students’ incomplete understanding to an already accepted equation.” Two of them (Barbara and Debbie) indicated that they have or plan to use the specific tasks from the videos. Barbara wrote, “I plan to use all the lessons which were introduced at the first of the year. Currently, our scope and sequence (standards) for 6th graders is working with pattern recognition (functions).” Charlene talked about how it will impact her teaching style, “I am going to try and implement Kirk’s teaching style as shown in the Growing Dots by allowing students to explain their answers and see where the gaps are, as well as allowing more classroom discussion.”

At the beginning of the module and then again approximately one month after the last session, the same class was observed for each of the 4 teachers. The observer used the structured observation tool and kept field notes. Each teacher was interviewed by the observer pre and post lesson. The lesson was given a synthesis rating using a 5-point scale (1 low-5 high) on design, implementation, mathematics content, and classroom culture/mathematical norms, and then given a capsule rating on one of 5 levels (from ineffective instruction to exemplary).

Two of the teachers’ (Arlene and Barbara) post observation occurred while they were using a task (Growing Dots) from one of the cases. Neither reported trying to do what they saw in the video, but rather use it for their own purposes. For example, Barbara indicated that they would be “working toward solving and recognizing one-step linear equations.” Charlene was teaching the “math support” class where students had a prescribed district curriculum to review and practice basic skills.

The results of the pre/post observations are contained in Table 3. Cells are highlighted where there was improvement from pre to post. Debbie and Barbara showed improvement in each category and the overall lesson, while little or no improvement appeared with Arlene and Charlene. A capsule rating of 2 indicates that there may be elements of effective instruction, but with *serious problems* in the design, implementation content and/or appropriateness for many students in the class. For example, in Arlene’s pre-observation the observer comments, “The mathematics in this lesson lacked rigor and was not at grade level… On the whole, although engaging for some students, the lesson was limited in its ability to enhance students’ understanding of the basics of place value.” For the same teacher in the post-observation, although she employed one of the tasks from the video the observer notes, “information about the Growing Dot Task was mostly ‘told’ to students, multiple representations and approaches

	Arlene		**Debbie**		**Barbara**		**Charlene**	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Design	2	2	2	3	2	3	2	2
Impl.	2	2	2	3	2	3	2	2
Content	1	2	1	2	1	2	1	1
Norms	1	1	1	2	1	2	1	1
Capsule	2	2	2	3	2	3	2	2

Table 3

were not encouraged, and student participation was limited to one word answers, without explanation. Mathematical connections were not made beyond vocabulary and definitions of the parts of the equation.

A capsule rating of 3 indicates beginning stages of effective instruction. Both Debbie and Barbara were rated a solid 3, which indicates instruction is purposeful and characterized by quite a few elements of effective practice. Students are, at times, engaged in meaningful work, but there are weaknesses, ranging from substantial to fairly minor. The observer noted about Debbie's lesson, "While there are elements of effective practice, some are better developed that others. … manipulatives to model and show multiple representations were made available for a teacher directed experience. The design of the lesson included working to find patterns for the different problem types and then generalizing the rules for integer addition. Students did make connections between use of the integer chips and the symbolic representation of the problems."

A comparison of Barbara's pre and post observer comments illustrate some of the differences in her two lessons:

> Pre - *Instruction appeared disconnected from the math content in that it did not appear to allow students to explore the patterns some were beginning to recognize. Directions to uncover the primes on a hundreds chart were given one at a time and without reference to skip counting or multiples."*
>
> Post - *Students were not encouraged to use the visual representation to discover what was changing/growing from diagram to diagram. ... Students primarily counted to fill in their t-chart. When the lesson moved to transferring the t-chart data to the graph, there was little or no connection to the visual representation, or the task itself; only the t-chart was referenced to provide the ordered pairs.*

Post 3-month interviews

Three of the four teachers were interviewed by telephone. Debbie was off track and not available at the time of the interview. They were sent copies of their pre and post responses on the external and embedded assessments and asked to look back over them prior to the interview. They were asked to comment on what they thought the pre and post assessments could and could not tell us about their experience with the video cases, what might they change, how the experience has impacted how they think about their teaching and what they think others might gain from participating in this series.

Despite repeated prompting, all 3 said little about the usefulness of the assessments. Barbara, for example said that she felt the instruments only partially demonstrated her understanding of how students learn this process. When asked why she indicated that she thought it was an issue of time. She needed time to reflect on the experiences and to try things in her classroom. Arlene indicated that she thought the assessments would tell others where she lacked in math and needed more support, but wasn't sure because she didn't get feedback as to correct answers. She indicated that she would have like to have spent time in the session going over the external assessment. The feedback would have made them more worthwhile to her. She noted that the questions made her think about things she hadn't considered since she graduated from college—it made her think about her own classroom.

All three teachers had praise for the sessions and reported that they learned a great deal. All reported that it was making a difference in their teaching. Charlene said it was "eye opening" to see kids have a voice versus teachers explaining. She hadn't seen anything like that before and it made her want to strive to do likewise. She wasn't in a teaching situation that allowed her to teach linear functions, but she talked about how the ideas "transferred" to other areas. She talked about a probability project she did and the videocase experience caused her to stop and look at the prior knowledge the kids brought to the project before moving forward with it.

Arlene said, "It brought so many things together for me. One huge evolution for me was that I was able to see how things all come together—tables, 2-step equations, graphing. … it dawned on me that I used to give two-step equations to students and never gave them the vocabulary to handle it." She said she used the growing dots task to introduce two-step equations this year so they could see the coefficient and the constant (this was a lesson observed). She indicated that her students could talk about the terms and relate them to what was changing and to where the line on a graph intersected the y-axis.

Barbara talked about how she is, "still working on learning" from the experience. She talked about her need for time to reflect on the ideas and work on them in her own classroom. She talked about the impact of her school situation. During this time she indicated that her school was being investigated by the state and this created a lot of stress. She said it may have seemed at times like she wasn't always "there" but she was. She said this meant, however, that she needed more time to "absorb" the ideas. She indicated that she now sees her own teaching as providing an ongoing opportunity for her to learn. She said she is trying activities with her students and using her experience with the videocases to help her "look at her students differently." When asked what about the professional development caused this she explained that she had participated in an extended workshop on functions 3 years earlier, and felt like she learned a lot, but was never able to apply anything to her teaching. The videocase experience provided incentive to try it in her classroom. She said it was due to the way the sessions were run, the facilitator, "didn't tell us what we should do—it wasn't like a classroom. Rather, it was the collaboration—people learning from one another." She said she thought that this was supported by the fact that the facilitator was genuinely interested in how they thought.

Analysis

The analyses consisted of passes through the various data. First, for each teacher, each of the data sources was examined for evidence of progress on the VCMPD goals for mathematical knowledge and professional practice. Next, each teacher's combined data was examined for patterns of growth related to the program goals. Finally, similarities and differences were identified among the four teachers in a comparative attempt to draw some tentative conclusions and raise questions.

Examining each teacher's growth

Arlene is a 7th grade mathematics teacher. During the study she was in her second year at this level in her school and was completing her forth year of teaching, having previously taught at the elementary level in another state. She had neither a major or minor in mathematics, but held a degree in education. She taught at what was known as the "worst performing school" in the district with a "challenging student population." She said she volunteered to attend the professional development because a district curriculum specialist had recommended it to her and was one of the most vocal participants from the onset.

Her analysis of teaching appeared to show progress toward module goals. Her post module video analysis provided a more detailed mathematical analysis of student thinking. Others have noted the importance of recognizing the details involved in understanding student thinking (Carpenter & Franke (2002); Sherin (2001). Her response to what the teacher might be thinking illustrates a shift from a definitive conclusion (The "teacher is trying to catch up with the amount of different perspectives and questions and address each.") to raising questions about the mathematics of reconciling differences ("Does the given equation fit James perspective? Is it the same as Danielle's or different?"). This, we see as evidence of a shift in attention to mathematical differences (Yackel & Cobb, 1996). Her suggestions for next steps in student work analysis near the end of the module, indicates that students could use help to develop understanding of the constant and the rate of change, and proposes the *Dots* task as a vehicle for doing this, ~~thus~~ recognizing how a task might be used to aid the development of the concept of rate of change. Her suggestions, however, do not go so far as to provide a rationale for how the task might be used to specifically help students see the relationship between the starting point and the constant in the expression (she doesn't appear to recognize the problem context can aide in developing the understanding of the relationship between the center dot, the constant and the beginning of the sequence). In the external assessment she displayed some improvement in her mathematics task performance, but no apparent change with regard to implications for instruction. Analysis of the videos of the group sessions revealed that as a member of the group she spoke frequently and displayed confidence and certainty in her verbal contributions during discussions. She often shared what she did or would do with her students. In the later sessions she did begin to ask her colleagues what they thought of her ideas and some degree of curiosity and tentativeness in her claims, "when James labeled the first dot 'x' I really wish we could have asked James why did you label the first dot 'x'? What is another way looking at it you could have named it?" This could suggest a shift to a more questioning attitude related to a classroom situation.

The classroom observations failed to demonstrate a significant change in her practice. In her post observation, she used a linear relationship task (*Dots* task) and in her interview said she valued visual approaches, but the observer found no evidence of students actually using them in the lesson. She improved slightly on the content synthesis rating. In her post-module interview, she reported that she learned that there are multiple approaches to solving problems and that seemingly simplistic approaches can lead to sophisticated conclusions. It appears that Arlene has gained an awareness and appreciation for various student approaches, but for some reason does not yet employ this in her practice.

Overall, Arlene made some slight gains mathematically that show up in differing data points but the experience appears to have made little impact on her teaching practice by the time of the last observation. Considering these data as a whole, Arlene's mathematical knowledge may have improved somewhat as well as her ability to recognize and consider teaching moves. The transfer to her own practice did not seemed to have occurred at this point.

Debbie teaches 6th grade. The study occurred during her third year of teaching. She holds a degree in mathematics as well as education. In the pre-survey she stated that she volunteered to participate in order to grow professionally and learn more strategies for the classroom.

Debbie's analysis of teaching and ideas related to instruction showed progress toward module goals. Her post video analysis provided a more detailed analysis of students—in particular she wrote about her interpretation of the distinctions between Brandie and James mathematical ideas. "Brandie interpreted the problem as what is the equation for the growing part of the dots, therefore, not adding in the 1. James not clear about variable or constant. He never came up with 4x, he just kept adding 4." After analyzing the students' work in session 7, she indicated that she would use a different model [from the stars] for the students to determine a general equation, she would use a chart and graph to help students see the linear pattern, and would emphasize the role of change and the constant. This could show an increased ability to prepare for the choice and use of various mathematical representations in developing students' conceptual understandings.

Debbie appears to show only a little improvement in her mathematical understanding—a net improvement of one on the external assessment. This is not surprising given her strong mathematics background. She did, however, show significant growth in her analysis related to instruction. She was able to provide a thoughtful rationale for her choices.

Debbie's teaching practice appeared to improve. Her classroom observation data showed a solid shift in all categories, from a capsule rating of 2 to a solid 3. The observer particularly noted her growth in the lesson design—even though the lesson in the post observation was not focused on algebra it reflected the use of multiple representations as well as the use of student ideas, including utilizing student errors to explore mathematical ideas. Debbie indicated in her post observation interview that she, "tried to group similar problems together hoping students would see the patterns, but I noticed there wasn't enough room on the board to display all of the problems." She said that she wanted to see more connections and would try to pull it all together in the next lesson.

At the end of the eight sessions, Debbie wrote that she learned that in teaching mathematics "it is important to allow students to discover solutions, give them time to find solutions, and pay attention to how students approach their solutions because usually there are many methods in which to solve a problem, whether it be closed, explicit, recursive, etc." Debbie appears to have gained in her ability to mathematically distinguish and categorize student methods.

Barbara teachers 6th grade and mathematics department chair at the same school as Arlene. During the course of the pilot the school was sanctioned by the state for poor performance of students and its principal was fired. She had been teaching a total of six years and holds a degree in education, but neither a major or minor in mathematics. Barbara indicated that she volunteered to attend the professional development experience in order to assist a new teacher on campus and because she "loves math" and was "excited about any and all information to expand her experience."

Her analysis of teaching and implications for instruction demonstrated progress toward module goals. She displayed increased detailed in the analysis of student thinking. In her analysis of Brandie and James (two students in the video) she made note of a teaching opportunity—to utilize their thinking by connecting the center dot and the concept of a constant term. Her choice of video segment in the pre video analysis focused on Danielle, whom she stated "understood function, how it effected growth of shape and clearly presented her idea." In the post, however, she chose to consider Brandie and James' focus on the center as a "teachable moment" for highlighting that "the center stays the same and is the constant." After analyzing student work in session 7, Barbara indicated that she would have students compare the difference between approaches in a more in-depth conversation. She also indicated that she would model different approaches next lesson incorporating graphing and emphasizing slope. Barbara appears to have gained in her ability to interpret and distinguish methods as well as value relating student approaches as a means for deepening mathematical discussions.

The video analysis of the sessions reveals that as a member of the group, Barbara appeared to be attentive to others ideas, but mostly quiet herself. In her post interview she noted this herself, indicating that she was really engaged and often contemplating ideas that had emerged. She also reported that her school situation sometimes left her preoccupied and stressed.

The classroom observation data indicated an improvement in her teaching practice—from a pre capsule rating of 2 to a solid 3 in the post. Several elements of the module goals were observed in her post observation. She showed improvement in all four of the observation categories: design, implementation, content and norms, even though she stated in the observation interview that "having to push through the curriculum due to school sanctions [makes] it difficult to find time for students to absorb the concepts." At the end of the module she wrote about her learning. "I learned that in teaching mathematics it is important to allow student to discover solutions, give them time to find the solutions. Also, pay close attention to how students approach their solutions – there are usually many methods (ways) in which to solve a problem, whether it be closed, explicit, [and] recursive." Like Debbie, Barbara appears to have gained in her ability to mathematically distinguish and categorize student methods.

Overall, Barbara made gains mathematically that showed up in both the external assessment measures and the classroom observation. She demonstrated improvement in her practice in the observation data, but it wasn't reflected in the external assessment with regards to implications for instruction. In her post module interview she talked about how she is still learning from the experience. In fact she said that she see how she can use her own teaching experiences as a way to keep learning. She attributed her growth to the way the sessions were run. She said that the facilitator didn't tell them what they should do—it wasn't like a classroom. Rather, it was the collaboration—people learning from one another. She said this was supported by the fact that the facilitator was genuinely interested in how they thought.

Charlene teaches 7th grade and serves as the department chair at the same school as Debbie. At the time of the study she had been teaching for twelve years. She has neither a major nor minor in mathematics, but holds a degree in education. In the pre-survey she stated why she volunteered to participate in the eight professional development sessions, "I never heard of anything like this before and thought it would be interesting if I could extend my professional growth in math."

Charlene showed only a little progress in her analysis of practice. In her pre-post video analysis she shifted from noting what James didn't know ("he didn't see the center dot as one") to what he did know ("the center is not growing—it's a constant"). Her post analysis was more detailed mathematically than her pre. In analyzing student work in session 7, she omitted any analysis—only reporting the percentage of students that got the correct answer.

Charlene demonstrated little progress toward module goals in terms of her own practice other than her own self-reports. The classroom observation data showed no observable impact on her teaching—she stayed the same in all categories—an overall capsule rating of two. This could be due, in part, to the nature of the class she taught—a low-level basic skills class. Using a prescribed set of skills materials may have precluded her opportunities to try new ideas in her classroom. The district mandated a tightly defined curriculum of arithmetic procedures. When asked what she learned at the end of the professional development experience, Charlene did display a subtle but significant shift in her thinking about students. She indicated that she learned that mathematics is represented and conceptually processed by individuals in many ways. She said that she was already aware of "different learning styles" of her students, but through this experience became more aware of the "differentiation in conceptualness." This appears to reflect a shift from thinking of student differences in terms of visual or auditory learners to an awareness of various ways of conceptualizing mathematics.

Drawing tentative conclusions and raising questions

Drawing some tentative conclusions from a cross analysis of the four teachers yields some interesting questions. All of the teachers showed an increased ability to provide a mathematically detailed analysis of a video of practice. In a follow-up study of CGI teachers, Franke and Carpenter (2001) describe teachers' growth in levels of engagement with children's mathematical thinking. The teacher development is described by benchmarks for each of the four levels. In their highest levels, 4A and 4B, one benchmark is "describes in detail individual children's mathematical thinking". Their findings are that the details involved in understanding student thinking are essential. We share this thinking and extend it to include the details of teaching as the Japanese do in their study of teaching (Yoshida 1999). To what extent this translates ultimately to improvement in practice is unclear. We did observe improvement in teaching with two of the teachers. Whether this is a true reflection of their learning from this experience remains to be seen. One pre/post lesson is insufficient to make any definitive conclusions, but the two teachers who demonstrated the most significant change in their teaching also displayed changes on the other measures as well.

All teachers reported that the experience was valuable and useful to their teaching practice. Each of them said that they were thinking about the different ways that students approach mathematical tasks and that they see the value of listening to and observing their students. Each of them reported that they are using these ideas in their teaching; however, evidence of this was only observed in two of the four.

Searching for explanations as to why two teachers appear to make significant progress toward module goals and two did not, yields more questions. Focusing first on external factors, could something like the number of years of teaching be a factor? The person with the most teaching experience (Charlene) showed the least progress, whereas the person with the least experience (Debbie with 3 years) demonstrated significant growth. Arlene, however, had only 4 years teaching experience and also demonstrated little progress. The opportunity to try things out in one's classroom may be a factor. Charlene, demonstrating the least improvement, was for the most part unable to try ideas with her students due to the prescribed basic skills sequence. Could mathematics background be a factor? Debbie, with her degree in mathematics, showed significant improvement in practice. The other three had no formal mathematics preparation and one of them (Barbara) still made significant progress. It should be noted that Debbie began at a high level on her mathematics pre assessment and had little room to improve in this regard, whereas Barbara also showed significant improvement in her mathematics. Could the school environment account for the changes? The teachers who appeared to demonstrate significant growth were from different schools. One of these two, Barbara, reported that her school context was most stressful, made progress in spite of these adverse conditions.

Looking at the materials themselves and the nature of the professional development experience, what accounts for the apparent growth of two of the teachers? They all report that the collaboration was important—being able to interact with and listen to one another's ideas. But many professional development experiences possess this element and yield little improvement in practice. What about the apparent improvement in mathematics? Sessions involved the teachers in working on the tasks from the videos, but in ways that differ from many professional development experiences. They searched for relationships across tasks and across representations. Visual approaches were emphasized. All of their mathematical work was couched in the context of real classrooms and they had opportunities to see how kids and teachers interacted with the tasks. What and how might this have had an impact? All of the teachers demonstrate growth in their ability and inclination to provide a more detailed mathematical analysis of teaching. How might the professional development have affected this?

A curious finding from these four teachers is that the two teachers who demonstrated growth were both relatively silent participants in the sessions. Historically facilitators have used vocal participation as evidence of engagement, which translates to a greater potential for learning. These data (although limited in scope) suggest something different. This creates a potential dilemma for facilitators of these materials if it is the case that vocal participation is not an indicator of learning. And to the contrary, the two teachers who fail to show growth were both vocal and could be judged to be quite engaged.

What we do not know is the impact of these experiences over time. Will their experiences ultimately prove to translate to the classroom? Will they be more capable of "seeing" and "hearing" their students? Will they be better equipped to predict student approaches and design tasks to accommodate this? Will they see their own classrooms as sites for learning? Certainly Barbara indicated that this was the case. What will it take to research these issues? How might future VCMPD pilot data analysis refute or back up these tentative findings? What if we observed these same teachers a year from now? What if these teachers were able to engage in more experiences like this?

Conclusions and Implications

This study was not intended to reach definitive conclusions about what or how teachers learn from the VCMPD materials, but the data does afford us the opportunity to identify a set of hypotheses for further investigation. As our analyses of these four teachers demonstrate, we are left with more questions than conclusions. We have received funding from the National Science Foundation[11] to conduct more thorough research on what teachers might learn from these materials.

The issues and ideas identified in this study will allow us to refine the hypotheses that form the basis of this new research effort. It is from that perspective that we put forth some tentative hypotheses we feel worthy of further investigation.

An interesting outcome from this limited study is that the teacher who had the least opportunity to examine the ideas in the context of her own classroom experienced the least growth. One hypothesis might involve investigation of the degree to which teachers must have opportunities to (be asked to) try ideas in their own classrooms throughout the professional development sessions. It might be worth considering the timing of sessions to allow for ample time between sessions to encourage more classroom investigation. The pilot sessions were spaced with only one week between sessions. How might the data have changed if sessions were spaced differently? Also worth considering is the degree to which there are specific classroom assignments that encourage paying more attention to student methods and thinking.

One of the two teachers (Barbara) who seemed to display considerable growth indicated in the post 3-month interview that she continues to benefit from the sessions. She said that she sees her classroom as a site for her ongoing learning. She indicated that this experience provided her "the incentive to try things" in her own classroom. She attributed this to the nature of the professional development sessions—the facilitator, "didn't tell us what to do. It wasn't like a classroom. She was genuinely interested in how we thought." A hypothesis to explore might be related to this "listening to participants" structure. To what degree does "listening to participants", trying to figure out what they bring and trying to determine what they know and understand, contribute to teachers' own ability to do this with their students?

Barbara's interview data reinforce the desirability of follow-up interviews and classroom observations several months or even years after the initial experience. What seeds were planted by this experience? How do the teachers make use of it and what further work might be beneficial? VCMPD has created several extension modules building from the 8-session

[11] In Spring 2003, "Turning to the Evidence" (#0231892) began a 3-year research study to examine what teachers learn by using classroom artifacts in professional development. The project is centered at EDC with the VCMPD component housed at WestEd.

foundation module. How would participation in one or more of those extensions benefit these teachers? Might those teachers not displaying growth benefit from more experiences and more time?

The data also suggest that perhaps the practice-based nature of the professional development materials may play a role in improving teachers' ability to think about their own practice. A hypothesis to consider is the degree to which artifacts of practice actually do ultimately promote improvement of practice.

This paper has examined data from four teachers to better understand what they might learn and use in their practice. As a preliminary study, we have attempted to use it to begin to define what data might be useful in a more rigorous effort to address these questions. It is clear that assessing teacher learning is a complicated effort, particularly trying to measure growth in content knowledge as well as their ability to use this knowledge in practice. To compound this problem, some of the assessments felt like tests to the teachers. If professional development is to help teachers improve their practice, however, we must improve and refine attempts to assess what teachers learn and apply in their practice.

References

Ball, D. L. (1991) Teaching mathematics for understanding: What do teachers need to know about subject matter? In M Kennedy (Ed), Teaching Academic Subjects to Diverse Learners New York: Teachers College Press.

Ball, D. L. and. Hyman, B., (2000). Interweaving content and pedagogy in teaching and learning to teach: knowing and using mathematics. In . J. Boaler. (Ed.), Multiple Perspectives on Mathematics Teaching and Learning Westport, Ct, Ablex Publishing**:** 83-104.

Barnett, C. S. and P. A. Tyson (1993). Case methods and teacher change: Shifting authority to build autonomy. Paper presented at annual conference of American Educational Research Association, Atlanta. GA.

Carpenter, T.P., Fennema, E., Peterson, P.I., & Carey, D.A. (1988). Teachers pedagogical content knowledge of students' problem solving in elementary arithmetic. Journal for Research in Mathematics Education, 19, 385-401.

Fenemma, E., & Franke, M.L. (1992). Teachers' knowledge and its impact. In Handbook of Research on Mathematics Teaching and Learning. D. G. (Ed.). New York, Macmillon**:** pp. 147-165.

Franke, M.L., Carpenter, T.P., Levi, L., & Fennema, E. (2000) Capturing teachers' generative growth: A follow-up study of professional development in mathematics. American Educational Research Journal, 38, 653-689.

Hill, H. & Collopy, R. (2003). Learning content in the context of practice: A videocase curriculum example. Paper prepared for the 2003 annual meeting of the American Educational Research Association, Chicago.

Hill, H. & Collopy, R. (2002). What might teachers learn: An evaluation of the potential opportunities to learn in Videocase . An evaluation report to VCMPD, University of Michigan

Hill, H. C., Schilling, S. & Ball, D. L. (2002) Developing Measures of Teachers' Content Knowledge for Teaching. Paper prepared for the 2002 annual meeting of the American Educational Research Association, New Orleans.

Lampert, M. (2001). Teaching Problems and the Problems of Teaching. New Haven: Yale University Press.

Ma, L. (1999). Knowing and Teaching Elementary Mathematics, Hillsdale, NJ: Lawrence Erlbaum Associates.

Schifter, D. (1998). Learning mathematics for teaching: From the teacher's seminar to the classroom. Journal for Mathematics Teacher Education **1**(1): 55-87.

Sherin, M. G. (2001). Developing a professional vision of classroom events. In T. Wood, B. S. Nelson, & J. Warfield (Eds.), Beyond classical pedagogy: Teaching elementary school mathematics. Hillsdale, NJ: Erlbaum.

Smith, M. S. et. al. (2003) Tracing the development of teachers' understanding of proportionality in a practice-based course. Paper prepared for the 2003 annual meeting of the American Educational Research Association, Chicago.

Stein, M. K., et. al (2003) What and how teachers learn from narrative cases: an example of teachers learning to productively use student responses. Paper prepared for the 2003 annual meeting of the American Educational Research Association, Chicago.

Tambe, P., St. John, M., Fuller, K., Houghton, N., Mitchell, J., Evans, T., (2002). Videocases for mathematics professional development: lessons learned from the pilot test. An evaluation report. Inverness, CA.

van Es, E., & Sherin, M. (2001). Learning to Notice: Scaffolding new teachers' interpretations of classroom interactions. Unpublished manuscript, Northwestern University. IL.

Wilson, S.M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: A review of research on contemporary professional development. Review of Research in Education, 24, 173-209

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematics Education, 27, 458-477.

Yoshida, M. (1999) Lesson Study [Jugyokenkyu] in elementary school mathematics in Japan: A case study. A paper presented at the American Educational Research Association 1999 Annual Meeting (April 19-23, 1999, Montreal, Canada).