LEARNING AND TEACHING LINEAR FUNCTIONS

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This paper reports on the methodological approach and preliminary findings from a research study of the Learning and Teaching Linear Functions professional development program. The study is currently in its third year and is in the beginning stages of data analysis. The Linear Functions for Teaching project investigates the research questions: (a) what do teachers learn from participating in the Learning and Teaching Linear Functions video-based professional development? And, (b) what do their students learn? Initial results will be shared regarding teacher learning of mathematical knowledge for teaching.

Keywords: Algebra and Algebraic Thinking, Mathematical Knowledge for Teaching, Teacher Education-Inservice/Professional Development, Teacher Knowledge

Introduction

Over the past two decades mathematics education research has identified ways in which more than a procedures-based approach to algebra is needed (Bednarz, Kieran, and Lee, 1996). For example, research has documented the various challenges students (and their teachers) face with the concept of slope. Students have difficulty interpreting linear functions and their graphs (Schoenfeld, Smith, & Arcavi, 1993), connecting graphs to linear equations (Kerslake, 1981), and connecting graphs to the notion of rate of change (Bell & Janvier, 1981). The mathematics education community has been working towards a goal of having algebra students explore relationships between symbolic expressions, graphs of lines, and to the meaning of intercept and slope. If that understanding is to occur, teachers must figure out how to use tasks and orchestrate discussions around linear functions to push for conceptualizing linear functions.

The Linear Functions for Teaching study focuses its work on the video case materials, *Learning and Teaching Linear Functions* (LTLF) (Seago et al., 2004), which are designed to enable teachers to deepen their understanding of ways to conceptualize and represent algebra content within their teaching practice. LTLF is premised on the idea that using artifacts of practice within a well-structured PD program can promote mathematical knowledge for teaching. This idea is supported by a variety of learner-centered, inquiry-based theoretical traditions, including constructivist and situative perspectives on learning (Cobb, 1994). These perspectives share the notion that engaging in challenging, problem-based, collaborative, and socially shared activities is likely to promote an expanded knowledge base (Borko, et al., 2005). The *Learning and Teaching Linear Functions* materials were designed with all of these features in mind and include an analytic framework, explicit tasks, teacher learning goals, and facilitation supports.\

Theoretical Framework

The theoretical frame for the LTLF video case materials is adapted from the work of Deborah Ball and colleagues (Cohen, Raudenbush & Ball, 2003) that incorporates research on both teaching and learning. The content of the video case materials focuses on the interactions between the teacher, the content (in this case, linear functions tasks), and the students, within the context of an authentic classroom environment (see Figure 1). The materials are designed to be used by a teacher educator who is faced with a similar set of relationships: the interactions between the teacher educator, the content (in this case, teaching and learning of linear functions), and the teachers he/she works with. To assist the teacher educator in using the PD materials productively with teachers, in-depth resource materials are provided to facilitate teachers' knowledge development. Resource materials include: mathematics content information, probing discussion questions, and other facilitation guidance specific to the materials.

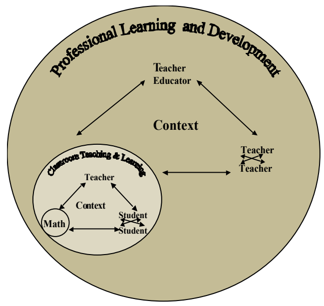

Figure 1: Theoretical Framework (Adapted from Cohen, Raudenbush & Ball, 2003)

As Ball and her colleagues have noted, of central importance with respect to interactions around the content with students is the teachers' mathematical knowledge for teaching (MKT; Ball, Hill & Bass, 2005). Their research has shown that MKT relates to the quality of teachers' classroom work and positively predicts gains in their students' mathematical achievement (Hill, 2010). MKT can be understood as the knowledge that teachers need to effectively carry out the work of teaching. MKT incorporates subject matter knowledge as well as pedagogical content knowledge.

The Learning and Teaching Linear Functions Video Case Materials

Learning and Teaching Linear Functions was designed to enable teachers to deepen their understanding of ways to conceptualize and represent linear functions within their teaching practice. The major goals of these materials are to help teachers deepen their understanding of mathematics content, students' mathematical thinking, and instructional strategies; as well as develop norms and practices for learning about teaching. The first module, *Conceptualizing and Representing Linear Relationships,* is a sequential series of eight 3-hour sessions that are designed to enrich teachers' ability to teach linear relationships and deepen their own detailed knowledge of the distinctions and linkages among the various representations. Each session has at its core one or two digital video clips of a mathematics classroom. These clips are unedited segments selected from real classroom footage of un-staged mathematics lessons, representing a range of grade levels, geographic locations and student populations.

Method

The Linear Functions for Teaching efficacy study consists of 66 teachers from across the state of California. A mixed within-school and between-school random assignment design was used. All potential participants were informed at the outset that they have a 50% chance of being assigned to the intervention group. All interested teachers were randomly assigned to treatment and control groups (34 treatment and 32 control group teachers). The study is currently in its third year and is in the beginning stages of data analysis. The LTLF foundation module was used with 34 treatment teachers in a 5-day institute August 2011. Two instruments were used to assess impacts of the LTLF PD on teachers' knowledge of mathematics for teaching: the Learning Mathematics for Teaching online instrument (TKAS) and the Artifact Analysis Assessment. Both of the assessment items include pre- and post-measures and were administered to both treatment and control teachers. The TKAS assessment focused on grades 6-8 patterns, functions and algebra scales. The Artifact Analysis Assessment consists of analyzing a linear functions task and solution methods; viewing a 5-minute video clip of 6th grade students presenting solutions to the same linear functions problem and then providing written responses to a series of increasingly detailed questions about the video. The assessment also asks teachers about three samples of student work on the problem (each represented a different kind of common student error). A scoring rubric was developed for participant responses based on a small set of randomly selected pre- and post-pilot data (blind scored. The rubric used to score this assessment includes three dimensions determined to be central to PD intervention goals:

- Justification: Making claims and providing evidence for them or making claims without providing evidence; and
- Interpretation: Focusing on students' potential understandings (vs. deficits in understanding);
- Attention: Focusing on specific mathematics content, generic mathematics content, or non-mathematical content.

Responses for each of the three areas were analyzed separately. We established inter-scorer reliability of 0.82 using percents of agreements among four researchers who used the coding scheme for scoring 34 teacher responses. We blind scored (treatment/control and pre-post) all responses in January 2012.

Preliminary Results

As stated earlier, we are in the initial stages of data analysis. Preliminary results of the impacts of the LTLF intervention on treatment teachers' mathematical knowledge for teaching are somewhat promising. Table 1 shows the TKAS pre-test scores by treatment/control status. Table 2 shows the post-test scores.

Table 1: Pre-test TKAS Scores by Treatment/Control Status

	Treatment M (SD)	Control M(SD)	Difference	p-value	Effect Size	Sample Size
LMT Scores (Pre-test)	0.36 (0.79)	0.53 (0.72)	-0.17	0.38	-0.23	63

Table 2: Post-test TKAS Scores by Treatment/Control Status

	Treatment M (SD)	Control M (SD)	Difference (SE)	p-value	Effect Size[B]	Sample Size
No Pre-test Adjustment						
LMT Scores (Post-test)	0.45 (0.91)	0.44 (0.70)	0.01	0.95	0.02	63
Pre-test Adjustment						
LMT Scores (Post-test)	0.53 (0.91)	0.35 (0.70)	0.18 (0.10)	0.09	0.26	63

Teachers in the treatment group exhibited growth of TKAS scores of 0.09, while control group teachers exhibited a 0.09 decline in scores. The difference in these two trajectories results in a net increase for the treatment group of 0.18 relative to the control group. This difference in change scores is not statistically significant at conventional levels, but is suggestive of a trend favoring the treatment group.

The Artifact Analysis assessment shows significant results for treatment teachers compared to control teachers. For the student video, student work, and teacher video, no statistically significant treatment/control differences on the pre-test measures were apparent. For the post-test measures, treatment teachers were substantially more likely to indicate an understanding of students' potential than control teachers on the student work task (47% vs. 7%). Treatment teachers were also more likely to focus on the mathematical content of student work (78% vs. 44%). There was also a tendency for treatment teachers to use evidence to justify their inferences with regard to student work (p =.12) and the teacher interview (p =.12), although these differences were not statistically significant at conventional levels.

Conclusion

Preliminary findings indicate that the treatment provided teachers with significant additional understandings in understanding students' potential, focus on the mathematical content of student work and use evidence to justify their inferences. The teacher learning exhibited in the early results is consistent with some of the learning goals of the LTLF PD intervention. The artifact analysis measure imitates most closely the work that teachers did within the PD (work on a math task, analyze a classroom video of students working on the task, and analyzing student work from the class). Other learning goals are subtle and may require more sensitive measures. As our early analysis emerges, we sometimes feel that the data from our measures capture only part of the story. It is important to find and use responsible measures that can (1) assess the impact of a PD intervention's goals for teaching learning, and (2) accurately measure the subtle and incremental nature that so often characterizes teacher learning. This provides us with a challenge for how to measure dimensions of teacher learning in ways that 1) allow comparisons across studies; 2) seem to capture the essence of the learning; and 3) respectfully characterize the complex and challenging work that teachers undertake when they engage in PD.

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